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Evidence for the Absence of Gluon Orbital Angular Momentum in the Nucleon *

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The Sivers mechanism for the single-spin asymmetry in unpolarized lepton scattering from a transversely polarized nucleon is driven by the orbital angular momentum carried by its quark and gluon constituents, combined with QCD final-state interactions. Both quark and gluon mechanisms can generate such a single-spin asymmetry, though only the quark mechanism can explain the small single-spin asymmetry measured by the COMPASS collaboration on the deuteron, suggesting the gluon mechanism is small relative to the quark mechanism. We detail empirical studies through which the gluon and quark orbital angular momentum contributions, quark-flavor by quark-flavor, can be elucidated.

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The nucleon is a composite particle with spin 1/2. There is little doubt that the theory of quantum chromodynamics (QCD) describes the manner in which the nucleon's spin is carried by its constituents, yet clarifying the details of this picture has incited intense theoretical and experimental activity [1]. Much has been made of the empirical fact that the spin of the nucleon is not given by the net helicity of its valence quarks [2]; however, this is not so much a "crisis" for QCD as it is for the non-relativistic quark model, since the latter rationalizes the charges, spins, and magnetic moments of the baryons in terms of the properties of its constituent quarks. The rich structures revealed through deeply inelastic scattering experiments on the proton [3] and through Drell-Yan production of massive lepton pairs with hadron beams and targets [4], compellingly demonstrate the limitations of such a simple picture.

The nucleon contains both quark and gluon components in QCD, so that its spin of 1/2 must follow from the sum of the spin and orbital angular momenta carried by these constituents:

$$\frac{1}{2} = L_q^{\rm net} + \frac{1}{2}\Delta\Sigma + L_g + \Delta g \,, \tag{1}$$

where we write $\Delta\Sigma$ for the net helicity of the quarks. Our decomposition is referenced to a polarization axis, so that $L_q^{\rm net}$ and L_g are the components of the orbital angular momentum, due to quark and gluon constituents, respectively, with respect to that axis. The decomposition is not unique [5]. In what follows, we will use the decomposition based on the angular momentum tensor in light-cone gauge, $A^+ = 0$ [5], so that the gluon constituents have physical polarization $S^z = \pm 1$, and there are no ghosts [6]. Our conclusions concerning the decomposition of the nucleon spin will thus be specific to light-cone gauge, giving us a natural connection to the physics of light-front wave functions [6], which are invariably defined in this gauge. We note that a manifestly gauge-invariant decomposition is also possible [7].

Light-front wave functions (LFWFs) enjoy many advantages: they are frame independent, and the spins of the constituents satisfy $J^z = \sum_{i=1}^{n} S_i^z + \sum_{i=1}^{n-1} L_i^z$ Fock state by Fock state for a polarization axis $\mathbf{z}$ — we emphasize that there are only $n-1$ internal orbital angular momenta for a given Fock state with n constituents. The LFWFs are the eigensolutions of the QCD Hamiltonian defined at fixed light-front time $\tau = t + z/c$. Indeed, LFWFs are the natural way to understand the structure of hadrons as probed through lepton scattering experiments. For example, the computation of the electromagnetic elastic form factors in the light-front formalism [8, 9] yields the insight [10] that the anomalous magnetic moment $\kappa \equiv (e/2M)F_2(0)$ is non-zero only if the quark Fock components carry non-zero transverse orbital angular momentum, i.e., if $\mathbf{S}_\perp \cdot \mathbf{L}_\perp^q \neq 0$. We neglect fundamental T violation, so that κ is real [11]. Since the proton's anomalous magnetic moment is nearly twice that of its Dirac magnetic moment, a "spin crisis" in DIS could have been altogether expected.

In this paper we will study constraints on the orbital angular momentum of the nucleon's constituents using the azimuthal single-spin asymmetries produced from a target polarized transverse to the reaction plane. Such asymmetries have been seen in a variety of reactions, although we focus on those observed in semi-inclusive deeply inelastic lepton scattering (SIDIS), as in, e.g., $\ell p^\uparrow \to \ell' \pi^\pm X$. In general, the single-spin asymmetry is proportional to the invariant form $\epsilon_{\mu\nu\sigma\tau} P^\mu S_p^\nu p_\pi^\sigma q^\tau$ where S_p, the nucleon spin, satisfies $S_p^2 = -1$ and $S_p \cdot P = 0$, where P is the nucleon momentum and $q = p_{l'} - p_l$ is the momentum transfer, with $Q^2 = -q^2$. The correlation is proportional to $\mathbf{S}_p \cdot \mathbf{p}_\pi \times \mathbf{q}$ in the target rest frame, and since it is of leading twist, it obeys Bjorken scaling [12]. This pseudo-T-odd correlation is engendered by final-state interactions (FSI) of the struck quark, and thus it does not reflect a fundamental violation of time-reversal invariance [12, 13]. The discrete symmetry transformations in the light-front formalism are studied in detail in Ref. [11].

The azimuthal single-spin asymmetry (SSA) for $\pi^\pm$ production in SIDIS from a unpolarized beam and transversely polarized target, is defined as [14]

$$
\begin{aligned}
A_{UT}^{\pi^\pm}(\phi, \phi_s) &\equiv \frac{1}{|\langle S_p \rangle|} \left(\frac{N_{\pi^\pm}^\uparrow(\phi, \phi_s) - N_{\pi^\pm}^\downarrow(\phi, \phi_s)}{N_{\pi^\pm}^\uparrow(\phi, \phi_s) + N_{\pi^\pm}^\downarrow(\phi, \phi_s)} \right) , \\
&\equiv A_{UT}^C \sin(\phi + \phi_S) + A_{UT}^S \sin(\phi - \phi_S) + ... , \qquad (2)
\end{aligned}
$$

where the "C" and "S" superscripts refer to the asymmetries generated by the Collins and Sivers effects, respectively, and the definition of the azimuthal angles ϕ and ϕ_S are as in Ref. [14]. In the Collins mechanism [15], the asymmetry is formed through the product of the transversity distribution and a pseudo-T-odd spin-dependent fragmentation function which describes the correlation of the transverse polarization of the struck quark with the transverse momentum of the produced hadron. In the Sivers mechanism [16], the asymmetry arises from the product of a pseudo-T-odd distribution function which describes the correlation of the transverse momentum of the struck quark with the transverse nucleon spin and a spin-independent fragmentation function [17, 18]. The asymmetries arising from the two mechanisms differ in their precise azimuthal angle dependence and can be separated as in Eq. (2) [14]. If the thrust [19] of the quark jet can be empirically determined, then the Sivers asymmetry can be measured directly, where we replace $\mathbf{p}_\pi$ with $\mathbf{p}_q$ [12]. We consider the Sivers effect exclusively.

As discussed in Ref. [12], a non-zero Sivers SSA follows from the interference of two amplitudes $\mathcal{M}[\gamma^* p(J_p^y) \to F]$ of differing nucleon spin $J_p^y = \pm 1/2$ which couple to the same final-state F. In particular, the quantum numbers of the struck quark are the same in each case. The polarization axis $\mathbf{y}$ is chosen transverse to the scattering plane. The two amplitudes must also differ in their strong phase to generate a non-zero SSA, so that the SSA is proportional to $\mathrm{Im}(\mathcal{M}[J_p^y = +1/2]^* \mathcal{M}[J_p^y = -1/2])$. The photon cannot flip the helicity of the struck quark, so that the two amplitudes differ by $|\Delta L^y| = 1$ to yield a non-zero result. The requisite matrix element is related, but not identical, to the matrix element which generates the anomalous magnetic moment [12]. In particular, the presence of a strong phase engendered by FSI is essential to generating a non-zero SSA.

The Sivers SSA A_{UT}^S is determined by the function $f_{1T}^{\perp q}(x, \mathbf{k}_\perp^2)$, which is subsumed in $f_{q/p\uparrow}(x, \mathbf{k}_\perp)$, the distribution of unpolarized quarks in a transversely polarized proton of spin S and mass M; we define [20]

$$\begin{aligned} f_{q/p\uparrow}(x, \mathbf{k}_\perp) &= f_1^q(x, \mathbf{k}_\perp^2) - f_{1T}^{\perp q}(x, \mathbf{k}_\perp^2)\frac{\epsilon^{\mu\nu\rho\sigma}P_\mu k_\nu S_\rho n_\sigma}{M(P\cdot n)} \\ &= f_1^q(x, \mathbf{k}_\perp^2) - f_{1T}^{\perp q}(x, \mathbf{k}_\perp^2)\frac{(\hat{\mathbf{P}}\times\mathbf{k}_\perp)\cdot\mathbf{S}}{M}\,, \end{aligned} \tag{3}$$

in a frame where $\hat{P}$ and n, an auxiliary lightlike vector, point in opposite directions. We thus have

$$A_{UT}^S = -\frac{2}{M}\frac{\langle \sum_q |\mathbf{k}_\perp| e_q^2 f_{1T}^{\perp q}(x, \mathbf{k}_\perp^2)\mathcal{D}(z, \mathbf{p}_\pi, \mathbf{k}_\perp)\sin^2(\phi - \phi_S)\rangle}{\langle \sum_q e_q^2 f_1^q(x, \mathbf{k}_\perp^2)\mathcal{D}(z, \mathbf{p}_\pi, \mathbf{k}_\perp)\rangle}\,, \tag{4}$$

where $\mathcal{D}(z, \mathbf{p}_\pi, \mathbf{k}_\perp)$ contains the fragmentation function $D_q^{\pi^\pm}(z, p_\pi)$ and $\langle\ldots\rangle$ refer to the appropriate angle and $\mathbf{k}_\perp$ integrals [21]. The quark electric charge e_q dependence follows since $f_{q/p\uparrow}(x, \mathbf{k}_\perp)$ comes from the square of the scattering amplitude. The function $f_{1T}^{\perp q}(x, \mathbf{k}_\perp^2)$ can be extracted from $f_{q/p\uparrow}$, which is defined in a gauge-invariant way via [22, 23]

$$\begin{aligned} f_{q/p\uparrow}(x, \mathbf{k}_\perp) &= \int \frac{d\xi^- d^2\boldsymbol{\xi}_\perp}{16\pi^3} e^{-ik^+\xi^- + i\mathbf{k}_\perp\cdot\boldsymbol{\xi}_\perp} \\ &\times \langle P|\overline{\psi}(\xi_i, \boldsymbol{\xi})[\infty, \boldsymbol{\infty}; \xi^-, \boldsymbol{\xi}_\perp]_C^\dagger \gamma^+ [\infty, \boldsymbol{\infty}; 0^-, \mathbf{0}_\perp]_C \psi(0, \mathbf{0}_\perp)|P\rangle\,, \end{aligned} \tag{5}$$

where $[\ldots]_C$ denote gauge links stretched in both light-like and transverse directions [22], capturing the final-state interactions necessary for the SSA. As we shall see, the latter imply that the Sivers function can only be related, rather than identical to, the matrix element for the anomalous magnetic moment.

The ΔL^y needed for a non-zero single-spin asymmetry can stem from two distinct physical sources: ΔL^y can arise from either quark *or* gluon degrees of freedom [24, 25]. In the first case the virtual photon strikes a quark in a Fock component of the nucleon's light-front wave function, whereas in the second case, the virtual photon fuses with a gluon in some $|qqqg...\rangle$ Fock state of the nucleon to produce a $q\overline{q}$ pair. Analogues of both mechanisms can also contribute to the nucleon's anomalous magnetic moment κ, as we shall detail. However, the empirical anomalous magnetic moments of the proton and neutron sum to nearly zero, suggesting that the gluon contribution to the anomalous magnetic moment is small. We shall argue that a similar cancellation observed in the SSA data from a deuteron target allows us to conclude that the gluon mechanism is small in this case as well. The two mechanisms we discuss are distinct, in part, because the virtual photon interacts with either an "intrinsic" quark, namely, a multiply-connected constituent of a multi-parton Fock state, or an "extrinsic" quark, produced as a member of a $q\overline{q}$ pair from photon-gluon fusion. The intrinsic quark carries an orbital angular momentum L_q, whereas the extrinsic quark carries, in part, the orbital angular momentum of the gluon constituent L_g which spawns it. We can similarly distinguish intrinsic from extrinsic gluons, so that an intrinsic gluon is also a multiply-connected constituent of a multi-parton Fock state. Our separation can be clouded by QCD evolution effects, for an extrinsic gluon spawned by the DGLAP (Dokshitzer-Gribov-Lipatov-Altarelli-Parisi) splitting $q \to qg$ can fuse with the virtual photon to generate extrinsic quarks and a SSA. This mechanism thus serves to dilute the correlation between the gluon dynamics and the intrinsic gluon orbital angular momentum contribution to the proton spin.

We emphasize that the two mechanisms, quark and gluon, differ in their isospin character and, indeed, that SSA data on the proton and deuteron can be used to distinguish them. We note that a large SSA for π^+ production from a transversely polarized proton has been observed by the HERMES collaboration [14]; however, when an analogous observable is measured by the COMPASS collaboration from a deuterium target [26], the SSA is consistent with *zero*. The polarization of the deuteron itself is used to define the spin correlation. The deuteron with spin $S_d^y = +1$ normal to the scattering plane has both nucleon spins aligned $S_p^y = +1/2$ and $S_n^y = +1/2$. Since the deuteron is a weakly bound state, the SSA from the deuteron is the sum of single-spin asymmetries (SSAs) for the proton and neutron to a very good approximation. The small SSA observed with a ^{2}H target is, in fact, natural if the matrix element is related to that of the anomalous magnetic moment — the empirical p and n anomalous magnetic moments sum to nearly zero. However, we shall show that this connection can only occur with the L_q mechanism; the isospin structure of the L_g mechanism is altogether distinct. In this regard whether the gluon-mediated SSA emerges from intrinsic or extrinsic gluons is without consequence. We note, in passing, that the use of other polarized nuclear targets can give empirical checks of these observations; for example, polarized ^{3}He offers an effective neutron target, up to "spin dilution" corrections of some 10% [27]. We shall now develop these ideas in detail.

To set the stage, we review the manner in which the anomalous magnetic moment of the nucleon is connected to the quark orbital angular momentum in the light-front formalism. Working in the interaction picture for the electromagnetic current $J^\mu(0)$ and the $q^+ = 0$ frame [8, 9], we have [10]

$$\kappa = -\sum_a \sum_j e_j \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \psi_a^*(x_i, \mathbf{k}_{\perp i}, \lambda_i) \mathbf{S}_\perp \cdot \mathbf{L}_\perp^{q_j} \psi_a(x_i, \mathbf{k}_{\perp i}, \lambda_i) \equiv \sum_q e_q a_q \,, \qquad (6)$$

where we write κ in units of $e/2M$ and define $\mathbf{S}_\perp \cdot \mathbf{L}_\perp^{q_j} \equiv (S_+ L_-^{q_j} + S_- L_+^{q_j})/2$ with $S_\pm = S_1 \pm i S_2$ and $L_\pm^{q_j} = \sum_{i\neq j} x_i(\partial/\partial k_{1i} \mp i\partial/\partial k_{2i})$ — the last sum is over quark flavor q. Consequently, the orbital angular momentum contribution $L_\pm^{q_j}$ associated with a struck quark j in Fock state a is not an independent variable, but, rather, is determined by the sum of the orbital angular momenta of *all* the spectator partons in that Fock state. This notion also gives rise to the vanishing anomalous *gravito-*

magnetic moment for composite systems, Fock state by Fock state [28]. Although we regard $\mathbf{L}_\perp^{q_j}$ as the transverse orbital angular momentum associated with the struck quark j, the explicit sum over $i \neq j$ makes it apparent that the transverse orbital angular momenta carried by gluon spectators implicitly contributes to its definition. Moreover, both quark and gluon contributions from the parent nucleon Fock state are captured by the matrix element of the $L_\perp^{q_j}$ operator, as the gluon can fluctuate to a $q\overline{q}$ pair, to which the photon can couple. It is these quark- and gluon-mediated contributions which we distinguish as the "quark" and "gluon" mechanisms. We note that the light-front formalism in $A^+ = 0$ gauge permits a simple kinematic operator representation of the $\mathbf{L}_z$ operator; this, in turn, permits $\mathbf{S}_\perp \cdot \mathbf{L}_\perp^{q_j}$ to act as a ladder operator which raises or lowers the value of $\mathbf{L}_z$ in this representation. In the last equality of Eq. (6) we subsume the Fock-state sum to define the contribution to the anomalous magnetic moment, quark flavor by quark flavor; the a_j are real. The phase-space integration is given by

$$\int [\mathrm{d}x]\,[\mathrm{d}^2\mathbf{k}_\perp] \equiv \sum_{\lambda_i,c_i,f_i} \left[\prod_{i=1}^{n} \left(\int\!\!\int \frac{\mathrm{d}x_i\, \mathrm{d}^2\mathbf{k}_{\perp i}}{2(2\pi)^3} \right) \right] 16\pi^3 \delta\left(1 - \sum_{i=1}^{n} x_i\right) \delta^{(2)}\left(\sum_{i=1}^{n} \mathbf{k}_{\perp i}\right) , \quad (7)$$

where n denotes the number of constituents in Fock state a, and we sum over the possible $\{\lambda_i\}$, $\{c_i\}$, and $\{f_i\}$ in state a. The summations are over all contributing Fock states a and struck constituent charges e_j; we refrain from including the constituents' color and flavor dependence in the arguments of the light-front wave function (LFWF) $\psi_a^{S_z}$, which we define in the $A^+ = 0$ gauge, with the principal-value prescription for singularities in k^+. We emphasize that both quark and gluon degrees of freedom in the nucleon's LFWF contribute to Eq. (6). Either an intrinsic or extrinsic gluon constituent in the nucleon can couple to a photon via a $q\overline{q}$ pair. The lowest-order effective $\gamma^* gg$ vertex, a contribution to the anomalous magnetic moment, is forbidden by C invariance, though the radiation of an extra gluon from the effective vertex removes this constraint and makes it finite. We note the gluon mechanism should contribute to κ_p and κ_n with equal weight, up to isospin-breaking effects, yet the empirical isoscalar magnetic moment of the nucleon, $\kappa_S \equiv (\kappa_n + \kappa_p)/2 = -0.06$, is numerically very small relative to $|\kappa_{n,p}|$ — suggesting that the gluon mechanism is itself small.

Returning to the Sivers function, we define

$$f_{q/p\uparrow}(\eta, \mathbf{l}_\perp) = \overline{u}(P, \lambda') \left[f_1^q(\eta, \mathbf{l}_\perp^2)\gamma^+ - f_{1T}^{\perp q}(\eta, \mathbf{l}_\perp^2) i\sigma^{+\alpha} \frac{\mathbf{l}_{\perp\,\alpha}}{M} \right] u(P, \lambda) , \quad (8)$$

where $u(P, \lambda)$ is a Dirac spinor associated with a spin-1/2 state of momentum P and helicity λ [29], with $\mathbf{y}$ the polarization axis — only the $\alpha = 1, 3$ matrix elements are nonzero. We identify $f_{q/p\uparrow}(\eta, \mathbf{l}_\perp)$ as the function $q(\eta, \mathbf{l}_\perp)$ of Ref. [22], where we work in leading twist and ignore all QCD evolution effects. Turning to the light-front formalism in $A^+ = 0$ gauge, with boundary conditions appropriate to SIDIS, a Fock component of the proton's LFWF has the form $\tilde{\psi}_a^{S_y} = \psi_a^{S_y}\exp(i\phi_a^{S_y})$; we emphasize that the LFWFs are complex in this case [22]. Note that we contrast the LFWF for a proton in isolation to that for a proton immersed in an external electromagnetic gauge field. For simplicity we assume the LFWFs differ only in the phase $\phi_a^{S_y}$. Working in the $q^+ = 0$ frame we thus identify

$$f_{1T}^{\perp q}(\eta, \mathbf{l}_\perp^2)\frac{l_1}{M} = -\frac{i}{2}\sum_a\sum_{j=1}^{n}\delta_{qq_j}\int[dx][d^2\mathbf{k}_\perp]\Big\{\psi_a^{\uparrow *}(x_i', \mathbf{k}_{\perp i}', \lambda_i)\psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i)$$
$$\times \,\mathrm{Im}\,\exp\Big(i(\phi^\downarrow - \phi^\uparrow)\Big) + \psi_a^{\downarrow *}(x_i', \mathbf{k}_{\perp i}', \lambda_i)\psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i)\mathrm{Im}\,\exp\Big(-i(\phi^\downarrow - \phi^\uparrow)\Big)\Big\}\,, \quad (9)$$

where $\mathbf{k}_{\perp j}' = \mathbf{k}_{\perp j} + \mathbf{l}_\perp$ and $x_j' = x_j + \eta$ for the struck constituent j and $\mathbf{k}_{\perp i}' = \mathbf{k}_{\perp i} - x_i\mathbf{l}_\perp/(1-x_j)$ and $x_i' = x_i[1-\eta/(1-x_j)]$ for each spectator i, where $i \neq j$. The existence of a term in $\mathbf{l}_\perp$ mandates not only orbital angular momentum [30] but also the imaginary parts in the right-hand side (RHS) of Eq. (9) — a FSI phase must be present to incur a SSA. We have suppressed the arguments of ϕ^{S_y} but assert that it depends on the magnitude and not the direction of $\mathbf{k}_{\perp i}^{(\prime)}$, so that the $|\Delta L^y| = 1$ structure of the matrix element entails the l_1 dependence, specifically that the RHS $\sim -(i/2)[(l_3 - il_1) - (l_3 + il_1)] = -l_1$, as found in explicit model calculations [12, 23, 31]. The Kronecker δ ensures that the struck quark is of flavor q. Following the development of Eq. (6) in Ref. [10], we find that $f_{1T}^{\perp q}(x, \mathbf{l}_\perp^2)$ can also be written in terms of the matrix element of a spin-orbit operator, if we consider the leading terms as $\mathbf{l}_\perp \to 0$:

$$\frac{f_{1T}^{\perp q}(\eta, 0)}{M} = \sum_a\sum_j\delta_{qq_j}\int[\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp]\frac{1}{2i(1-x_j)}\Big[\tilde{\psi}_a^{*}(x_i', \mathbf{k}_{\perp i}, \lambda_i)\overline{\mathbf{S}}_{\perp T}\cdot\overline{\mathbf{L}}_{\perp T}^{q_j}\tilde{\psi}_a(x_i, \mathbf{k}_{\perp i}, \lambda_i)$$
$$-\overline{\psi}_a^{*}(x_i', \mathbf{k}_{\perp i}, \lambda_i)\overline{\mathbf{S}}_{\perp T}\cdot\overline{\mathbf{L}}_{\perp T}^{q_j}\overline{\psi}_a(x_i, \mathbf{k}_{\perp i}, \lambda_i)\Big]\,, \quad (10)$$

where $\overline{\psi}_a^{S_y} \equiv \psi_a^{S_y}\exp(-i\phi_a^{S_y})$, $\overline{\mathbf{S}}_{\perp T}\cdot\overline{\mathbf{L}}_{\perp T}^{q_j} \equiv (S_{+T}L_{-T}^{q_j} - S_{-T}L_{+T}^{q_j})/2$, $S_{\pm T} = S_3 \pm iS_1$, and $L_{\pm T}^{q_j} = \sum_{i\neq j} x_i(\partial/\partial k_{3i} \mp i\partial/\partial k_{1i})$. We define

$$\frac{f_{1T}^{\perp q}(\eta, \mathbf{l}_\perp^2)}{M} \equiv -\tilde{a}_q(\eta, \mathbf{l}_\perp^2)\,, \quad (11)$$

where the $\tilde{a}_j$ are real. A comparison of Eqs. (6) and (10) prompts us to include a minus sign in the definition of $\tilde{a}_q(x, \mathbf{l}_\perp^2)$. We note in passing that the generalized parton distribution $E(x,\zeta,t)$ probed in virtual Compton scattering (VCS) can also be connected to the nucleon's orbital angular momentum. The generalized form factors in VCS, $\gamma^*(q) + p(P) \to \gamma^*(q') + p(P')$ with $t = \Delta^2$ and $\Delta = P - P' = (\zeta P^+, \mathbf{\Delta}_\perp, (t + \mathbf{\Delta}_\perp^2)/\zeta P^+)$, have been constructed in the light-front formalism [33]. Using Eq. (40) of Ref. [33], and the procedure and syntax of Eq. (6), noting $\mathbf{q}_\perp \to \mathbf{\Delta}_\perp$, we determine, for $\zeta \leq x \leq 1$,

$$\begin{aligned} \frac{E(x,\zeta,0)}{2M} &= \sum_a (\sqrt{1-\zeta})^{1-n} \sum_j \delta(x-x_j) \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \\ &\times \psi_a^*(x_i', \mathbf{k}_{\perp i}, \lambda_i) \mathbf{S}_\perp \cdot \mathbf{L}_\perp^{\mathbf{q_j}} \psi_a(x_i, \mathbf{k}_{\perp i}, \lambda_i) \,, \end{aligned} \tag{12}$$

with $x_j' = (x_j - \zeta)/(1-\zeta)$ for the struck parton j and $x_i' = x_i/(1-\zeta)$ for the spectator parton i. We emphasize that the LFWFs for Fock-state a with spin up or down for fixed struck quark helicity differ by $|\Delta L^z| = 1$ because $\mathbf{L}_\perp^{q_j}$ contains ladder operators.

The SSAs and the Anomalous Magnetic Moments

We see that the matrix element, Eq. (10), which drives the Sivers SSA bears similarity to that of the anomalous magnetic moment [12, 34, 35, 32, 36, 37, 38]. To understand the consequences of this in a transparent way, we recall that under an assumption of isospin symmetry, we have $a_d^p = a_u^n$, $a_u^p = a_d^n$, $a_{\bar{d}}^p = a_{\bar{u}}^n$, $a_{\bar{u}}^p = a_{\bar{d}}^n$, and $a_{\bar{q}}^p = a_{\bar{q}}^n$ for sea quarks of other flavors. Isospin symmetry acts at the level of the hadronic matrix elements; the quark charges are not isospin mirrors. Neglecting the contributions of anti-quarks we have

$$\begin{aligned} &\kappa_p = 1.79 = (+2/3)a_u^p + (-1/3)a_d^p \\ &\kappa_n = -1.91 = (-1/3)a_d^n + (+2/3)a_u^n = (+2/3)a_d^p + (-1/3)a_u^p \,, \end{aligned} \tag{13}$$

to yield $a_d^p = a_u^n = -2.03$ and $a_u^p = a_d^n = 1.67$. Concommitant isospin relations follow for the $\tilde{a}_q$ of Eq. (11) as well. We also neglect the contributions of the anti-quarks to the existing SSA data, which is consistent with recent fits [21, 39, 40]. In what follows, if we *conjecture* that the isospin structure of the empirical anomalous magnetic moments is that of the Sivers SSA, then we find the relative strength of the various $\tilde{a}_q^p(\eta, \mathbf{l}_\perp^2)$ can be estimated through that of the a_q^p. Recalling Eqs. (4) and (11), the negative sign of a_d^p predicts a negative sign of the SSA for $lp \to l'\pi^- X$, whereas $a_u^p = 0.835$ predicts a positive asymmetry for mesons produced by favored fragmentation from the u quark. As noted in Refs. [35, 32, 36], both predictions are consistent with HERMES data at sufficiently large z [14].

As to the magnitudes of the SSAs, although $|a_u^p| < |a_d^p|$, the SSA engendered by u-quark fragmentation from a proton is enhanced by a factor of 4, since the asymmetry is controlled by e_u^2. In the absence of anti-quark contributions, the SSA for π^- production should indeed be small, and this is observed [14]. Recent model fits [21, 39] are consistent with these trends. To be more specific, we note the fits of Ref. [39] yield $S_u = -0.81 \pm 0.07$ and $S_d = 1.86 \pm 0.28$, where S_q is to be compared to $-\langle \tilde{a}_q \rangle$, with $\langle \tilde{a}_q \rangle$ defined to be the average value of $\tilde{a}_q(\eta, \mathbf{l}_\perp^2)$. This implies that the SSA asymmetry from u-quark fragmentation is smaller than that predicated from the use of the empirical anomalous magnetic moments alone. Although we expect the strong phase from the Wilson line to be isoscalar, the extra factor of $1/(1-x_j)$ in the matrix element of Eq. (10) could change the relative strength of the u and d contributions. Nevertheless, considering the consequences of this simple picture for the deuteron data, we note that $a_u^p + a_u^n = a_d^p + a_d^n = -0.360$, implying that the SSA for u-quark fragmentation to leading positively charged hadrons, as well as d-quark fragmentation to leading negatively charged hadrons, ought be small. This is borne out by the recent COMPASS data in SIDIS from a deuteron target [26]. Such a cancellation is consequent to the differing signs of $a_u^{p,n}$ and $a_d^{p,n}$ [21, 39].

Quark or Gluon Orbital Angular Momentum?

We now wish to use the differing isospin structure of the L_q and L_g mechanisms to infer the relative size of these contributions to the total orbital angular momentum of the nucleon. The isospin structure of the L_g mechanism is distinctive, for a gluon in a nucleon Fock state will produce $u\overline{u}$ or $d\overline{d}$ pairs with equal weight — up to tiny differences driven by the u-d mass difference. Thus the u-quark and d-quark SSAs add constructively in SIDIS from a ^{2}H target; there is no cancellation of this $I=0$ physics. However, the L_g mechanism cannot always generate a leading hadron, i.e., a contribution which survives in the $z \to 1$ limit. For example, to realize the $lp \to l'\pi^\pm X$ reaction via the L_g mechanism, *two* $q\overline{q}$ pairs must be produced to recover a $\pi^\pm$ hadron. It is significant, then, that the SSAs from the deuteron are consistent with zero for $z > 0.35$ [26], for both positively and negatively charged leading hadrons [41]. In this kinematic region the SSA asymmetry in π^+ production on the proton is increasing, so that the gluon mechanism can contribute in this z region. Moreover, the value of x, for leading hadrons, ranges from 0.006 to 0.3, so that the gluon mechanism can contribute in this x region. The empirical SSAs in leading hadrons from ^{2}H are consistent with zero, though, for all but the smallest z. In the context of the L_q mechanism this can be understood as emerging from an approximate cancellation of the p and n SSAs, reflective of the isospin structure of the anomalous magnetic moments. The ^{2}H data thus allow us to conclude that L_g is small compared to the quark contributions. We note in passing that theoretical arguments based on the large N_c expansion lead to a similar conclusion [42].

We can also crudely quantify the extent to which the L_g mechanism is absent. To estimate the relative size of the strong phase in the L_q and L_g cases, we employ the same reasoning as used in interpreting the ratio of the rapidity plateaus in gluon versus quark jets [43, 44, 45]. That is, we assume the phases scale, gluon to quark, as 2.25, as per a leading-order analysis in the QCD coupling [43, 44, 45]. Then, if we compute the ratio of the SSA from positively charged, leading hadrons on a ^{2}H target [26] to that from leading π^+ production on a proton target [14], and divide by a factor of roughly $2 \cdot 2.25 \approx 4.5$, as the deuteron contains two nucleons, we can estimate the relative strength of the two mechanisms. We find that the gluon mechanism is smaller than the quark mechanism by a factor of 0.2. The nucleon's orbital angular momentum appears to be largely carried by its quarks. We can also use the ratio of SSAs for u- to d-quark jets in SIDIS on the proton to determine the ratio of $\tilde{a}_u^p/\tilde{a}_d^p$ as a function of η and $\mathbf{l}_\perp^2$. To realize this experimentally, one needs to work at large z where the jet tagging is reliable, i.e., where the hadron type tags the flavor of the "struck" quark.

We have argued that the L_g mechanism is small and cannot always produce a leading hadron, so that one is left to ponder how current empirical constraints can be bettered. It strikes us as efficacious to study SSAs associated with produced hadrons of non-valence quark content. The $\gamma^* g \to s\bar{s} \to K^- K^+ + X$ reaction is one such possibility. In principle, one can trace the SSA of the $K^- K^+$ to the gluon's orbital angular momentum L_g. One can also consider the $\gamma^* g \to s\bar{s} \to \phi + X$ reaction: the SSA in ϕ production. Both reactions are important tests for the L_g mechanism, since the gluon contributions of the two nucleons to the SSA add. One can consider these processes as aspects of the gluon jet. In this, we ignore the possibility of intrinsic strangeness in the nucleon's non-perturbative structure, since parity-violating electron scattering experiments show the strangeness contribution to the proton's anomalous magnetic moment to be small [46]. We note that Anselmino et al. have discussed accessing the Sivers gluon distribution through open charm production [25]; this is similar in conception to the suggestions we offer here.

The Sivers asymmetry from gluons can also be studied directly, if the empirical thrust [19] of the gluon jet axis can be determined. If this could be done, the study of the L_g mechanism would be much facilitated, as the Collins mechanism would no longer contribute. To extract detailed numerical information about the gluon mechanism from the SSAs, one ultimately requires information about the strong phase from FSI; nevertheless, the experiments we suggest do serve to bound the size of the gluon's orbital angular momentum contribution to the nucleon's spin.

Let us conclude with a brief summary.

- A non-zero SSA follows from the interference of two amplitudes of differing nucleon spin, but of common quark helicity.

- The two amplitudes must also differ in their strong phase to generate a non-zero SSA, so that the matrix element which yields the Sivers function cannot be identical to that for the anomalous magnetic moment. Nevertheless, the signs of the SSAs from leading u- and d-quark fragmentation from the proton correlate with those of the u and d-quark contributions to the anomalous magnetic moment, analyzed under an assumption of isospin symmetry.

- The anomalous magnetic moment, the Sivers function, and the generalized parton distribution E can all be connected to matrix elements involving the orbital angular momentum of the nucleon's constituents. To be specific, the matrix elements are between LFWFs that differ by one unit of orbital angular momentum along the polarization axis; this follows as we compute matrix elements of the angular momentum lowering or raising operator.

- The SSA can be generated by either a quark or gluon mechanism, and the isospin structure of the two mechanisms is distinct. The approximate cancellation of the SSA measured on a deuterium target suggests that the gluon mechanism, and thus the orbital angular momentums carried by gluons in the nucleon, is small.

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Note added: After the completion of our paper, we became aware of work contemporary to ours by Anselmino *et al.*, Ref. [47], which draws similar conclusions from the measured SSA in $p^{\uparrow}p \to \pi^0 X$ scattering.

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